II. Solution by S. A. COREY, Hiteman, Iowa.

We have $1/y=\int_0^\infty e^{-ay}da$ if $y>0$. Hence

$$\int_0^\infty \frac{\sin my}{y}dy=\int_0^\infty \sin my\int_0^\infty e^{-ay}da\,dy=\int_0^\infty\int_0^\infty \sin my\, e^{-ay}\,da\,dy,$$

$$=\int_0^\infty\int_0^\infty \sin my\, e^{-ay}\,dy\,da=\int_0^\infty \frac{mda}{a^2+m^2}$$

$$=\tfrac{1}{2}\pi \text{ if } m>0,\ =-\tfrac{1}{2}\pi \text{ if } m<0,\ =0 \text{ if } m=0.$$

(See Byerly's *Integral Calculus*, page 100.) Similarly,

$$\int_0^\infty \frac{\cos my}{y}dy=\int_0^\infty \cos my\int_0^\infty e^{-ay}\,da\,dy=\int_0^\infty \frac{ada}{a^2+m^2}=0,$$

for all finite real values of m.

III. Remark by F. D. POSEY, A. B., San Mateo, Cal.

A discussion of the first of these integrals will be found on page 271, Art. 285 of Todhunter's *Integral Calculus*. The result there obtained is $\frac{1}{2}\pi$ if m be positive, $-\frac{1}{2}\pi$ if m be negative.

Also solved by G. W. Greenwood, G. B. M. Zerr, and J. Scheffer.

MISCELLANEOUS.

146. Proposed by F. P. MATZ, Ph. D., Sc. D.

Given $\left\{\begin{array}{l} a\cos\alpha+b\sin\alpha=c \\ a\cos\beta+b\sin\beta=c\end{array}\right\}$ to prove that

$$\sin(\alpha+\beta)=\frac{2ab}{a^2+b^2},\ \text{and}\ \cot\alpha+\cot\beta=\frac{2ab}{c^2-a^2}.$$

I. Solution by M. E. GRABER, A. M., Heidelberg University, Tiffin, Ohio.

We have at once $\dfrac{\cos\alpha-\cos\beta}{\sin\alpha-\sin\beta}=-b/a$ ....(1).

Now $\cos\alpha-\cos\beta=-2\sin\frac{1}{2}(\alpha+\beta)\sin\frac{1}{2}(\alpha-\beta)$ and $\sin\alpha-\sin\beta=2\cos\frac{1}{2}(\alpha+\beta)\times\sin\frac{1}{2}(\alpha-\beta)$. Substituting these values in (1) we obtain $\tan\frac{1}{2}(\alpha+\beta)=b/a$. Since $\tan\frac{1}{2}(\alpha+\beta)=\pm\sqrt{\dfrac{1-\cos(\alpha+\beta)}{1+\cos(\alpha+\beta)}}=b/a$, and solving,

$$\cos(\alpha+\beta)=\frac{a^2-b^2}{a^2+b^2}\text{.........(2)};\ \sin(\alpha+\beta)=\frac{2ab}{a^2+b^2}\text{.........(3)}.$$

Multiplying together the given equation and substituting the values of $\cos(\alpha+\beta)$,

$\sin(\alpha+\beta)$ from (2) and (3) we find that $\sin\alpha\sin\beta=\dfrac{c^2-a^2}{a^2+b^2}$; and therefore

$$\cot\alpha+\cot\beta=\frac{\sin(\alpha+\beta)}{\sin\alpha\sin\beta}=\frac{2ab}{c^2-a^2}.$$

II. **Solution by L. S. SHIVELY, Mt. Morris College, Mt. Morris, Ill.**

Subtracting the second equation from the first, $a(\cos\alpha-\cos\beta)+b(\sin\alpha-\sin\beta)=0$. Hence, $\sin\frac{1}{2}(\alpha-\beta)[a\sin\frac{1}{2}(\alpha+\beta)-b\cos\frac{1}{2}(\alpha+\beta)]=0$. Either $\sin\frac{1}{2}(\alpha-\beta)=0$ or $a\sin\frac{1}{2}(\alpha+\beta)=b\cos\frac{1}{2}(\alpha+\beta)$. In the first case we have the trivial solution $\alpha=\beta$. In the second case, we put $(\alpha+\beta)=x$, $\sin\frac{1}{2}x=b/a\cos\frac{1}{2}x$. Since $\sin x=2\sin\frac{1}{2}x\cos\frac{1}{2}x$ the latter equation takes the form $(a^2+b^2)\sin^2x=2ab\sin x$, whence, excluding the value $\sin x=0$ introduced by squaring, $\sin(\alpha+\beta)=\dfrac{2ab}{a^2+b^2}$.

Secondly, $\cot\alpha+\cot\beta=\dfrac{\sin(\alpha+\beta)}{\sin\alpha\sin\beta}$. From the first of the given equations $a\sqrt{(1-\sin^2\alpha)}+b\sin\alpha=c$. Solving for $\sin\alpha$, we have $\sin\alpha=\dfrac{bc\pm a\sqrt{(a^2+b^2-c^2)}}{a^2+b^2}$

Similarly, solving the second of the given equations, $\sin\beta=\dfrac{bc\pm a\sqrt{(a^2+b^2-c^2)}}{a^2+b^2}$

Since the solution $\alpha=\beta$ was rejected, $\sin\alpha\sin\beta=\dfrac{c^2-a^2}{a^2+b^2}$.

Hence $\cot\alpha+\cot\beta=\dfrac{\sin(\alpha+\beta)}{\sin\alpha\sin\beta}$ becomes equal to $\dfrac{2ab}{c^2-a^2}$.

III. **Solution by A. H. HOLMES, Brunswick, Maine.**

From $a\cos\alpha+b\sin\alpha=c$,

$$\sin\alpha=\frac{bc+a\sqrt{(a^2+b^2-c^2)}}{a^2+b^2},\quad \cos\alpha=\frac{ac-b\sqrt{(a^2+b^2-c^2)}}{a^2+b^2},$$

and from $a\cos\beta+b\sin\beta=c$,

$$\sin\beta=\frac{bc-a\sqrt{(a^2+b^2-c^2)}}{a^2+b^2},\quad \cos\beta=\frac{ac+b\sqrt{(a^2+b^2-c^2)}}{a^2+b^2}.$$

$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\sin\beta\cos\alpha$. Putting for $\sin\alpha$, $\cos\alpha$, $\sin\beta$, and $\cos\beta$ their values above, and reducing, $\sin(\alpha+\beta)=\dfrac{2ab}{a^2+b^2}$, $\cot\alpha+\cot\beta=\dfrac{\cos\alpha}{\sin\alpha}+\dfrac{\cos\beta}{\sin\beta}$.

Introducing the values of these sines and cosines, and reducing, $\cot\alpha+\cot\beta=\dfrac{2ab}{c^2-a^2}$.

Also solved by G. B. M. Zerr, G. W. Greenwood, J. Scheffer, and E. L. Rich.